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INVESTIGATION OF THE NATURAL FREQUENCIES OF FLUIDS IN SPHERICAL AND CYLINDRICAL TANKS

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INVESTIGATION OF THE NATURAL FREQUENCIES OF FLUIDS IN SPHERICAL AND CYLINDRICAL TANKS

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SUMMARY

Several small models of propellant-tank configurations applicable to missile designs were oscillated to study the natural frequencies of contained fluids over a range of fluid depths and tank sizes. The configurations included spheres and right circular cylinders which could be oriented with respect to the direction of oscillation.

The data are presented in terms of nondimensional parameters suggested by theoretical considerations. The experimental results obtained on tanks of various sizes indicate that these nondimensional parameters can be used to predict the natural frequencies of fluids in all tanks of a given configuration.

Comparison of the experimental results of this investigation with available theoretical results shows excellent agreement.

INTRODUCTION

The effects of the movement of large fluid masses on the stability of their containers or the overall vehicle are a matter of common experience. They have long been a factor in the stability of ships, have become of concern in the aircraft field, and are now of importance in missile operations. Fluid dynamic studies relative to the aircraft problem, such as those of references 1 to 6, were conducted to form a better understanding of the liquid motions inherent in airplane tip tanks. These studies were primarily concerned with finding ways to represent or approximate the complicated motions of fuels in oscillating tanks and with the determination of the effects of these motions on the flutter of wing-tank configurations.

Relative to missiles and spacecraft the problems associated with liquid propellants in motion are of increased significance during the launch stage due to the fact that the fuel and oxidizer for liquid-propellant boosters represent a large percentage of the total weight.

The oscillation of these free-surface propellant masses exert forces and moments on the missile which may couple with the control system and cause instability. These fluid oscillations, resulting from such sources as programed control pulses, have been shown experimentally (ref. 7, for example) to be most critical when the excitation frequency is in the region of a natural frequency of lower mode fluid oscillations. In addition, the frequencies of fluid oscillation may coincide with the frequencies of structural modes and induce resonances leading to amplified structural deformations. A knowledge of the natural frequencies of the liquids in fuel tanks of various configurations affords the designer information necessary to minimize possible resonant conditions.

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Analytical methods for the determination of the natural frequencies of contained fluids have been derived for special cases in available literature (refs. 1, 8, 9, and 10, for example) but experimental verifications of these analyses are heretofore limited to variations of upright cylindrical tanks and the first mode of spherical tanks.

The purpose of this paper is to report the results of an experimental investigation of the natural frequencies and mode shapes of fluids contained in representative missile fuel-tank configurations. The natural-frequency data obtained from fluids in spherical and cylindrical tanks of different sizes, fullness, and orientation with respect to the direction of oscillation are presented in nondimensional form and are compared with available analytical results.

SYMBOLS

g acceleration due to gravity

h liquid depth

l cylinder length

n mode of fluid oscillation

R cylinder or sphere radius

γ_n frequency parameter for nth mode for longitudinal modes of horizontal circular cylinders (fluid surface parallel to cylinder axis), $\omega_n\sqrt{\frac{l}{g}\,\frac{1}{\tanh\frac{n\pi h}{l}}}$

δ_n frequency parameter for nth mode for transverse modes of upright circular cylinders (fluid surface normal to cylinder axis), $\omega_n\sqrt{\frac{R}{g}\frac{1}{\tanh\frac{h}{R}\epsilon_n}}$

ϵ_n nth zero of first derivative of Bessel function of first order and first kind

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λ_n frequency parameter for nth mode for spheres and transverse modes of horizontal circular cylinders (fluid surface parallel to cylinder axis), $\omega_n\sqrt{\frac{R}{g}}$

ω_n experimental natural frequency of oscillation of nth mode

Ω_n analytical natural frequency of oscillation of nth mode

APPARATUS AND TEST PROCEDURE

Apparatus

Description of models.- The models of propellant-tank configurations studied in these tests include three spheres and four right circular cylinders capable of being oriented in different ways with respect to the direction of oscillation. The dimensions of the models are given in table I together with the corresponding ranges of liquid depths studied for each model. In the case of the upright circular cylinder, the table describes models of that configuration which were studied and discussed in references 7, 12, 13, and 14. The configurations for the circular cylinder are defined by the orientation of the cylinder with respect to the oscillation as illustrated in the table. The transverse and longitudinal modes of horizontal cylinders were obtained with the cylinder positioned so that the fluid surface was parallel to the cylinder axis and the transverse modes for upright cylinders were obtained with the cylinder oriented so that the fluid surface was normal to the cylinder axis.

All models were constructed of clear Plexiglas to permit visual observation of the fluid motion. In all cases, water was used as the fluid.

Mechanical shaker.- The models were mounted on a platform that was supported in pendulum fashion from overhead beams. Oscillation of the models was obtained by connecting the platform directly to a mechanical shaker as shown in figure 1. The mechanical shaker, described fully in reference 11, is essentially a slider-crank mechanism driven by a variable-speed motor. This apparatus was so designed as to provide a means for conveniently varying the frequency and amplitude of the reciprocating motion applied to the platform.

Test Procedure

The testing technique involved excitation of models over a range of frequencies to obtain the natural frequencies of the liquid. The procedure was repeated over a wide range of fluid depths. In measuring the frequencies of the lower modes, the mode in question was induced by the mechanical shaker, and upon full development of the wave form the tank motions were stopped and the frequencies were obtained by visually timing the low amplitude oscillations of the liquid during the decay of the wave form. In the case of the higher modes, the excitation amplitudes were maintained at low levels and the natural frequencies were taken as those frequencies yielding maximum fluid response. In this case, the frequencies were read directly from a tachometer, visible in figure 1. Figures 2 to 5 show typical shapes of the lower liquid modes in spherical and cylindrical tanks. Data were taken for all liquid modes visually detected with sufficient clarity for their definition.

DATA REDUCTION AND PRESENTATION

The natural frequencies measured for each model were nondimensionalized to provide relationships valid for different size models of a given geometrical configuration excited in a specified manner. In order to accomplish this, a frequency parameter was developed for each configuration from consideration of the variables involved. In the case of spheres and horizontal circular cylinders subjected to transverse oscillations, this parameter is the ratio of the experimentally determined natural frequency to appropriate variables used in the nondimensional treatment given these configurations in reference 8. No explicit expression for Ω_n is known for these configurations. The analytical values obtained in reference 8 correspond to the eigenvalues obtained from a matrix iteration scheme. The dimensionless parameter for the nth mode is denoted by λ_n and defined by the expression

$$\lambda_n = \omega_n\sqrt{\frac{R}{g}}$$

where ω_n is the experimental natural frequency of the nth mode, R is the cylinder or sphere radius, and g is the acceleration due to gravity.

The parameter developed for the natural frequencies of the liquid in horizontal cylinders subjected to longitudinal excitation is the ratio of the experimentally determined natural frequency to variables present in an analytical expression for the natural frequency of fluids in a rectangular tank (no known appropriate expression exists for circular tanks) as derived in reference 9. The expression for the natural frequency of the fluid is

$$\Omega_n = \sqrt{\frac{gn\pi}{l} \tanh \frac{n\pi h}{l}}$$

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where n is the mode of fluid oscillation, l is the cylinder length, and h is the liquid depth. The resulting parameter, denoted by γ_n, is

$$\gamma_n = \omega_n \sqrt{\frac{l}{g} \frac{1}{\tanh \frac{n\pi h}{l}}}$$

In the case of the upright circular cylinder, an exact expression exists for the natural fluid frequencies (ref. 9). This expression is

$$\Omega_n = \sqrt{\frac{g}{R} \epsilon_n \tanh \frac{h}{R} \epsilon_n}$$

where ϵ_n is the nth zero of the first derivative of the Bessel function of the first order and the first kind. The parameter δ_n, again the ratio of the experimentally determined natural frequency to the variables in the previous analytical expression, is defined by

$$\delta_n = \omega_n \sqrt{\frac{R}{g} \frac{1}{\tanh \frac{h}{R} \epsilon_n}}$$

These frequency parameters are plotted as a function of fluid depth $h/2R$ for each mode of the given configuration and presented in figures 6 to 9. The slight scattering of points noted in some cases at the high and low depths may be partially attributed to the difficulty involved in determining the exact frequency of the mode in question at those depths. Furthermore, the data obtained for the higher modes of the small models were limited because these mode shapes could not be clearly defined in some cases.

DISCUSSION OF RESULTS

Modes for Spheres

The experimental data for the three spheres are presented in figure 6 in terms of the frequency parameter. The measured values of the frequency parameter λ_n are given as a function of fluid depth for the first four liquid modes with curves faired through these points to differentiate between the modes. The figure indicates that at a given depth, or fixed value of $h/2R$, the frequency parameter for a given mode is the same for all models examined. It appears therefore that λ_n is independent of size.

It is interesting to note the variation of λ_n with depth for the modes considered. Note that, since λ_n for a given model is simply the product of a constant $\sqrt{\frac{R}{g}}$ and the natural frequency ω_n, these variations with depth are indicative of the natural frequency trend. In the first or fundamental mode, λ_n increases with fluid depth with a minimum at the near-empty condition. However, for the higher modes λ_n does not follow this monotonic trend, but instead the minimum value of λ_n appears to occur for the approximately half-full condition.

As the depth approaches zero, the value of the frequency parameter for a given mode approaches a finite value. In contrast, as the values of $h/2R$ approach unity (full tank), the value of the frequency parameter for each mode appears to become infinite. The finite values of λ_n under near-empty conditions may be interpreted physically as the result of the fluid acting as a compound pendulum for the case of the first mode and a multiple pendulum for the higher modes, the center of rotation being the center of the sphere. That λ_n should approach infinity at the higher depths may be physically interpreted as the result of the reduction in the free-surface area as the tank is filled. The values of these parameters at the end conditions and midpoint agree with the analytical values of reference 8. These analytical values, computed by an integral-equation approach, constitute the theoretical points presented in the figure.

Transverse Modes for Horizontal Circular Cylinders

Frequency-depth relationships for horizontal circular cylinders subjected to transverse oscillations are shown in figure 7 for four circular cylinders of different sizes. The frequency parameter λ_n is the same as that for the spherical case.

The results again appear to be independent of tank radii and applicable for predicting the natural fluid frequencies in any similar configuration. The figure also indicates that the transverse natural frequencies of fluids in horizontal cylinders are, as expected, independent of tank length. The general trend of the curves and the physical explanation of the boundary values are similar to the case of the spherical configuration. The theoretical curve was again obtained from the analytical results of reference 8 and, as the figure shows, excellent agreement with the experimental data is obtained.

Longitudinal Modes for Horizontal Circular Cylinders

The results obtained from longitudinal excitation of the four horizontal circular cylinders are presented in figure 8. The frequency parameter γ_n is shown as a function of depth for the first four liquid modes. As in the case of the sphere and transverse modes for the horizontal circular cylinders, the data obtained for a given longitudinal mode of the circular cylinders define one curve. It appears therefore that the resulting curve is applicable for all size cylinders although no theoretical verification is available for this case.

Since the resulting curve for each mode is essentially the product of a constant and the ratio of the experimentally determined natural frequency to the analytical expression for the natural frequency of a fluid contained in a rectangular tank, the variation of γ_n with depth represents the limitations involved in the analytical expression when applied to longitudinal modes of circular cylinders.

Transverse Modes for Upright Circular Cylinders

In order to illustrate experimentally the validity of the exact expression for the natural frequencies of liquids in upright circular cylinders undergoing transverse oscillations, experimental data are presented in figure 9 in a form similar to that used for the previous configurations. The frequency parameter δ_n is plotted as a function of fluid depth $h/2R$ for the first three liquid modes. Experimental data obtained from the excitation of one small tank during this investigation are supplemented by experimental data taken from tests presented in references 7, 12, 13, and 14. The theoretical values of the frequency parameter δ_n are also presented. These values, equal to $\sqrt{\epsilon_n}$, show excellent agreement with the experimental results obtained over a large range of tank sizes including full-scale missile tanks.

CONCLUDING REMARKS

Fluid frequency parameters are developed by relating experimentally determined natural fluid frequencies to certain physical parameters for several models of missile propellant tanks of different sizes. These parameters are found to be independent of container size and are believed to be applicable for tanks of all sizes up to and including full scale.

The data show that the characteristic variations of the natural frequencies with tank fullness vary considerably with tank orientation and mode of fluid oscillation. In the case of the transverse oscillations of fluids in spheres and horizontal cylinders, a monotonic increase in frequency from a finite value is noted for the first mode as the tank is filled. For the higher modes, the frequencies decrease from finite values for the empty condition to minimum values when the tank is approximately half full. As the tank is filled further, the frequencies again increase. As the full condition is approached, the frequencies of all modes approach infinite values. Excellent agreement is shown between the experimental data and the results of available theory.

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In the case of longitudinal oscillations of fluids in horizontal circular cylinders, the natural frequencies of all modes increase monotonically with tank fullness. No theoretical results are available for comparison for this configuration.

The analytical expression for the natural frequencies of a fluid undergoing transverse oscillations in an upright circular cylinder is found to yield results in close agreement with the experimental data obtained over a wide range of fluid depths and container sizes.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Field, Va., January 12, 1960.

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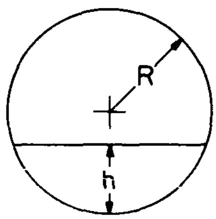
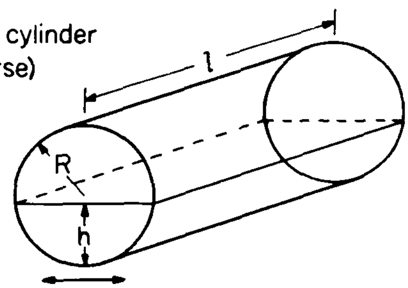
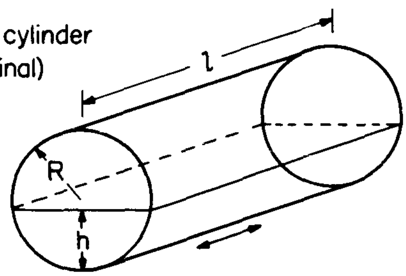
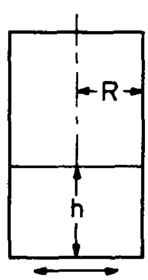
TABLE I.- DIMENSIONS OF TEST CONFIGURATIONS AND CORRESPONDING LIQUID DEPTHS STUDIED

Configuration	Model	R, in.	l, in.	h/2R
Sphere 	1	3.10		.048 - .904
	2	6.43		.070 - .955
	3	13.13		.019 - .914
Circular cylinder (transverse) 	1	2.75	16	.115 - .900
	2	4.28	15	.040 - .949
	3	6	12	.042 - .667
	4	6	24	.042 - .833
Circular cylinder (longitudinal) 	1	2.75	16	.091 - .909
	2	4.28	15	.058 - .967
	3	6	12	.083 - .833
	4	6	24	.083 - .833
Circular cylinder (upright) 	1	6		.215 - 1.745
	Ref. 7	5		2
	"	8.75		1.143
	"	12.5		.80
	Ref. 12	7.08		.505 - .785
	Ref. 13	7.08		.25 - .50
	Ref. 14	7.08		.50 - 1.0

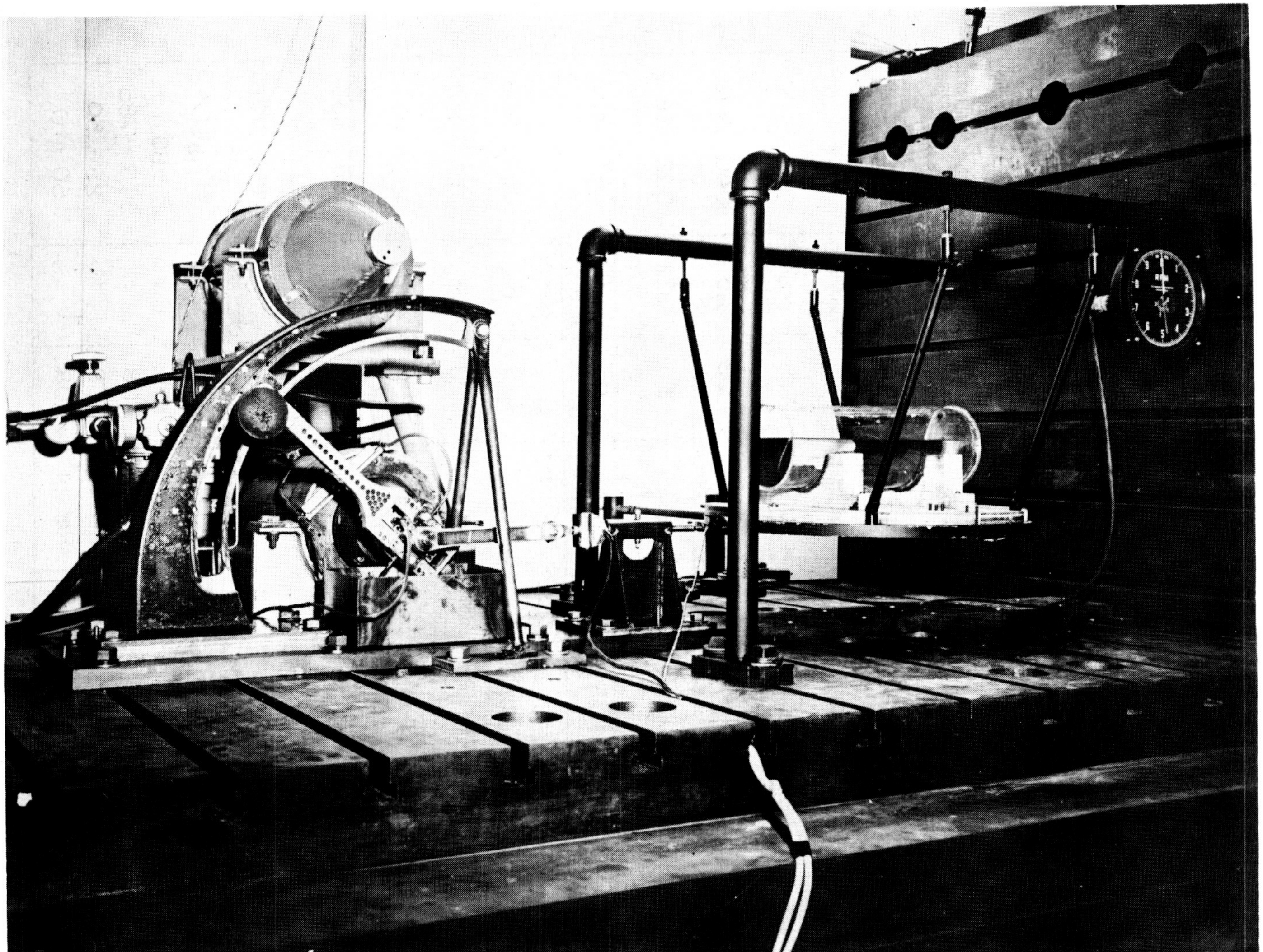


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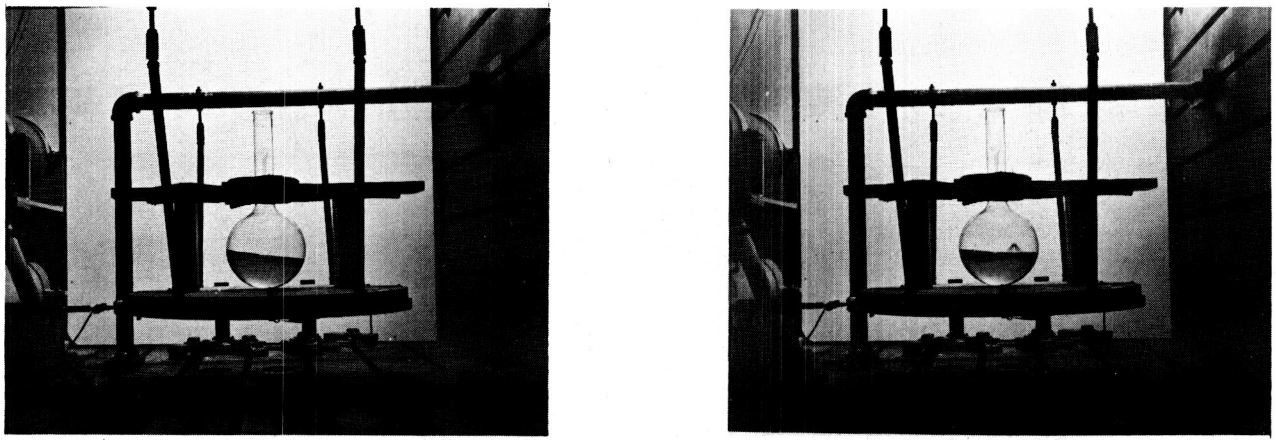
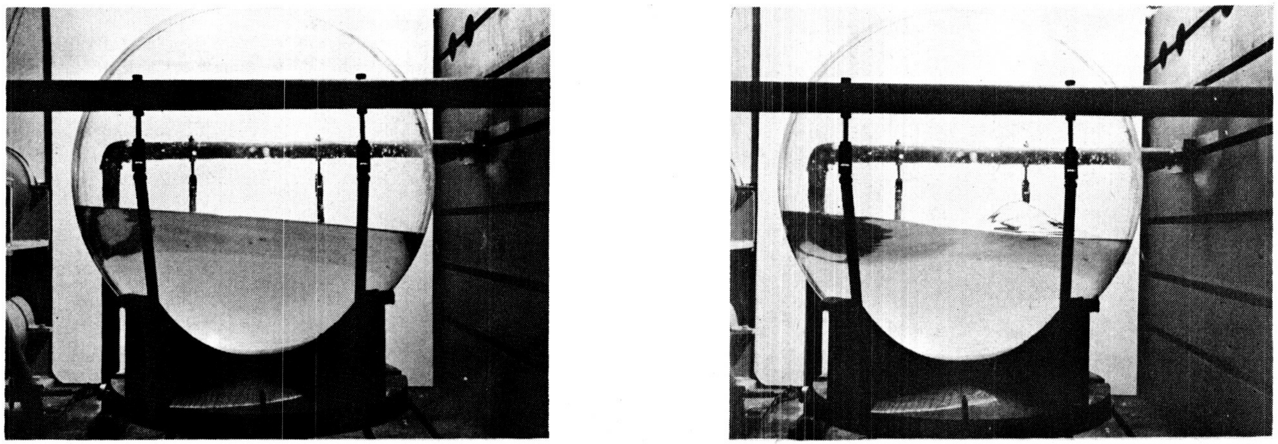
Figure 1.- Test apparatus for mechanically exciting liquid-propellant tank configurations.



(a) R = 3.10 inches.



(b) R = 13.13 inches.

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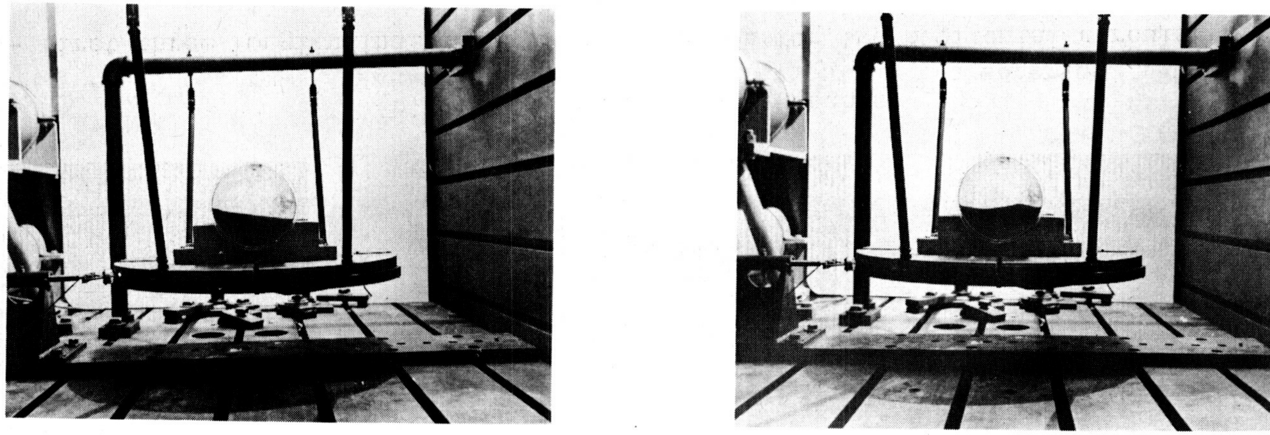
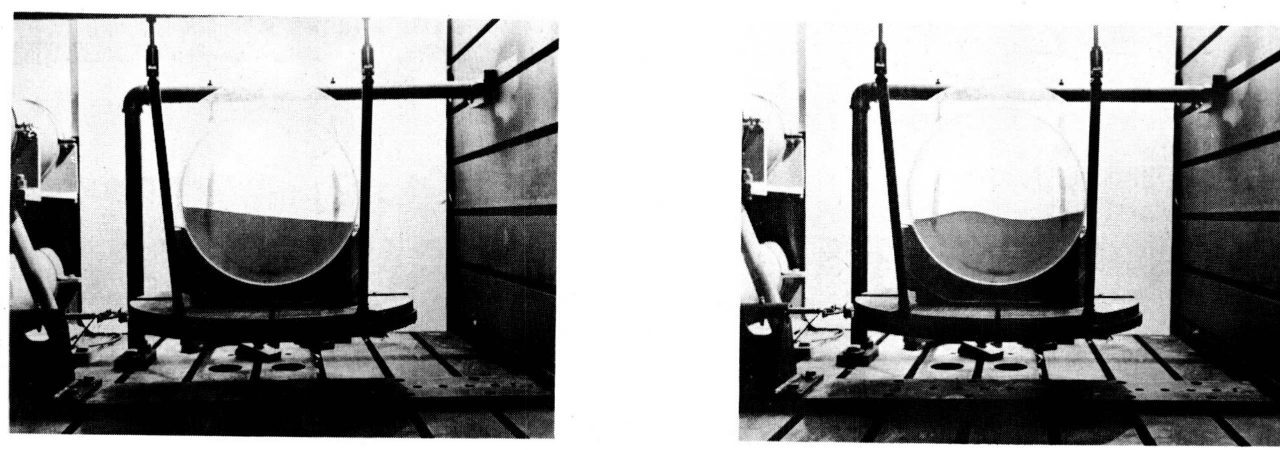
Figure 2.- First and second modes of oscillation for two spherical tanks.

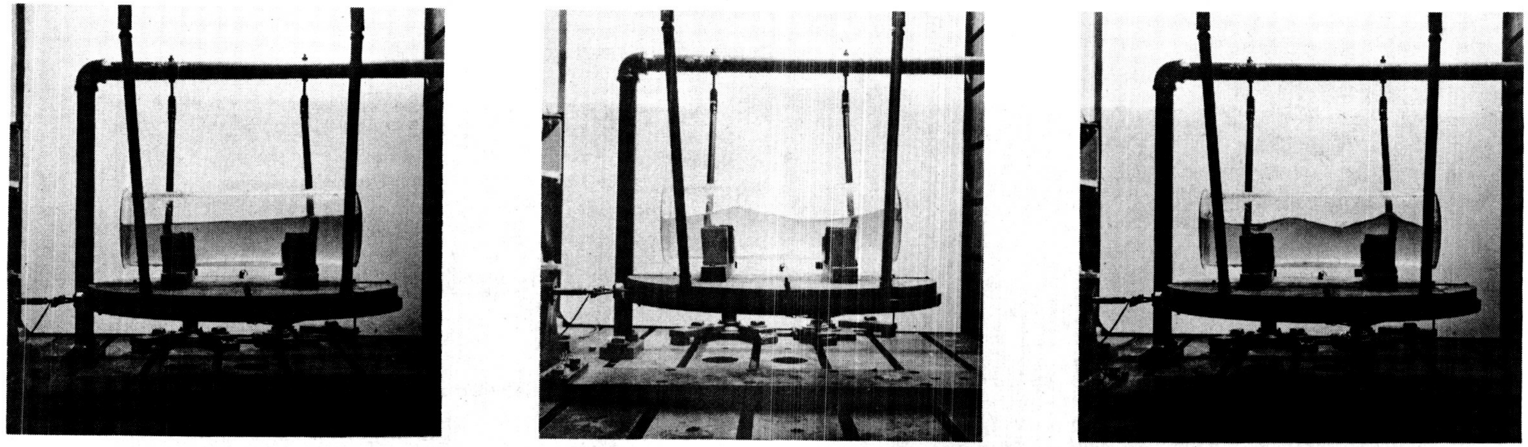


(a) R = 2.75 inches; l = 16 inches.

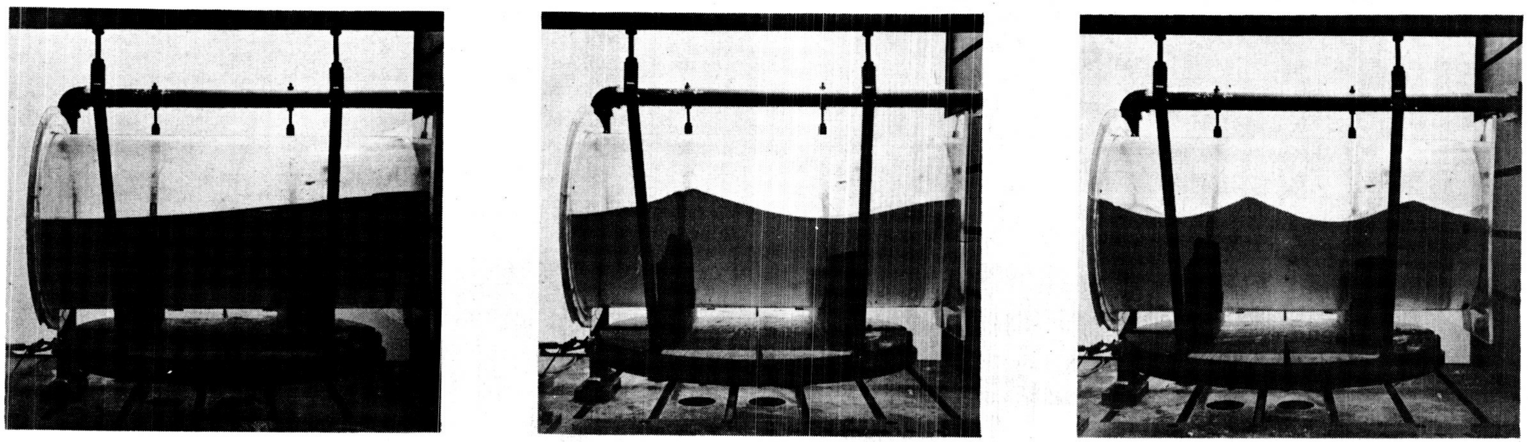


(b) R = 6 inches; l = 24 inches. L-60-204

Figure 3.- First two transverse modes of oscillation for two horizontal circular cylinders.



(a) $R = 2.75$ inches; $l = 16$ inches.



(b) $R = 6$ inches; $l = 24$ inches.

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Figure 4.- First three longitudinal modes of oscillation for two horizontal circular cylinders.

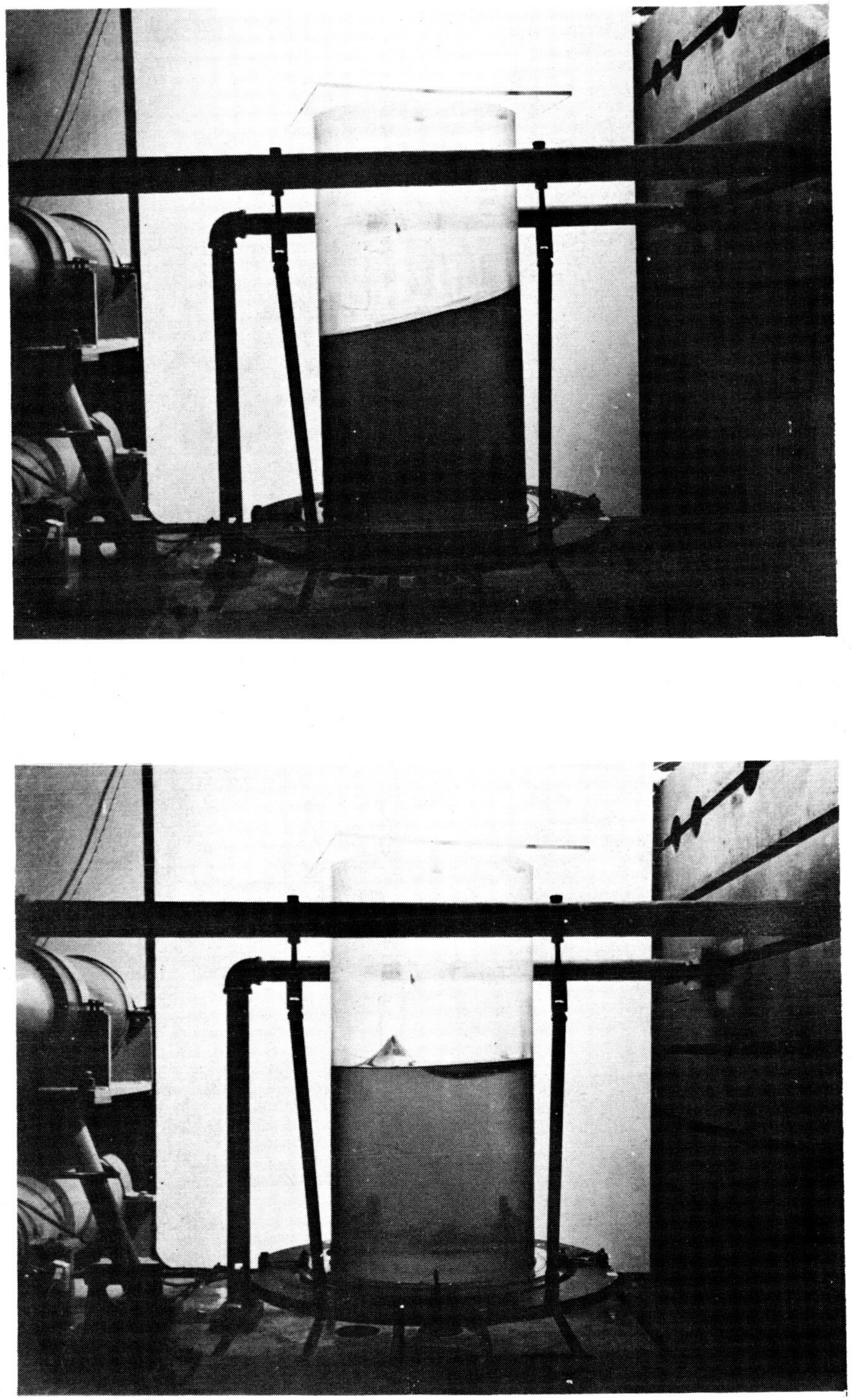

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Figure 5.- First two modes of oscillation for upright circular cylinders.

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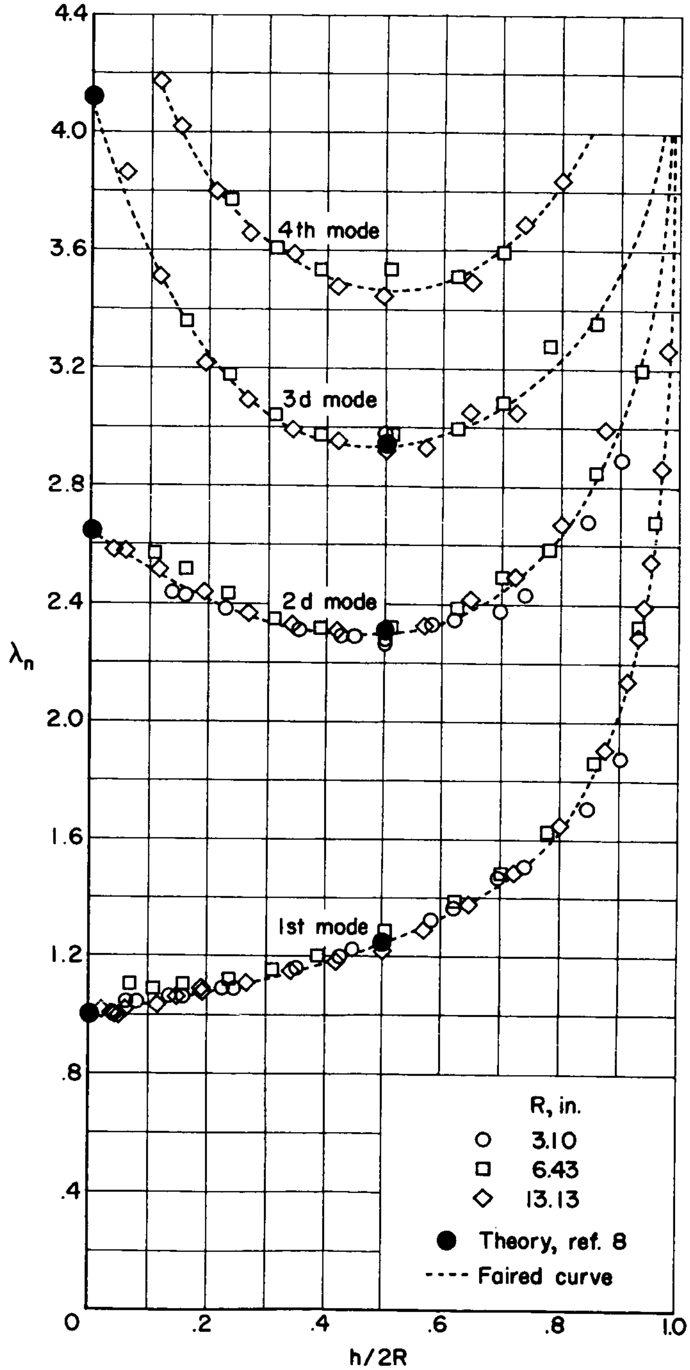

Figure 6.- Variation of fluid frequency parameter $\left(\lambda_n = \omega_n\sqrt{\frac{R}{g}}\right)$ with depth for spherical tanks.

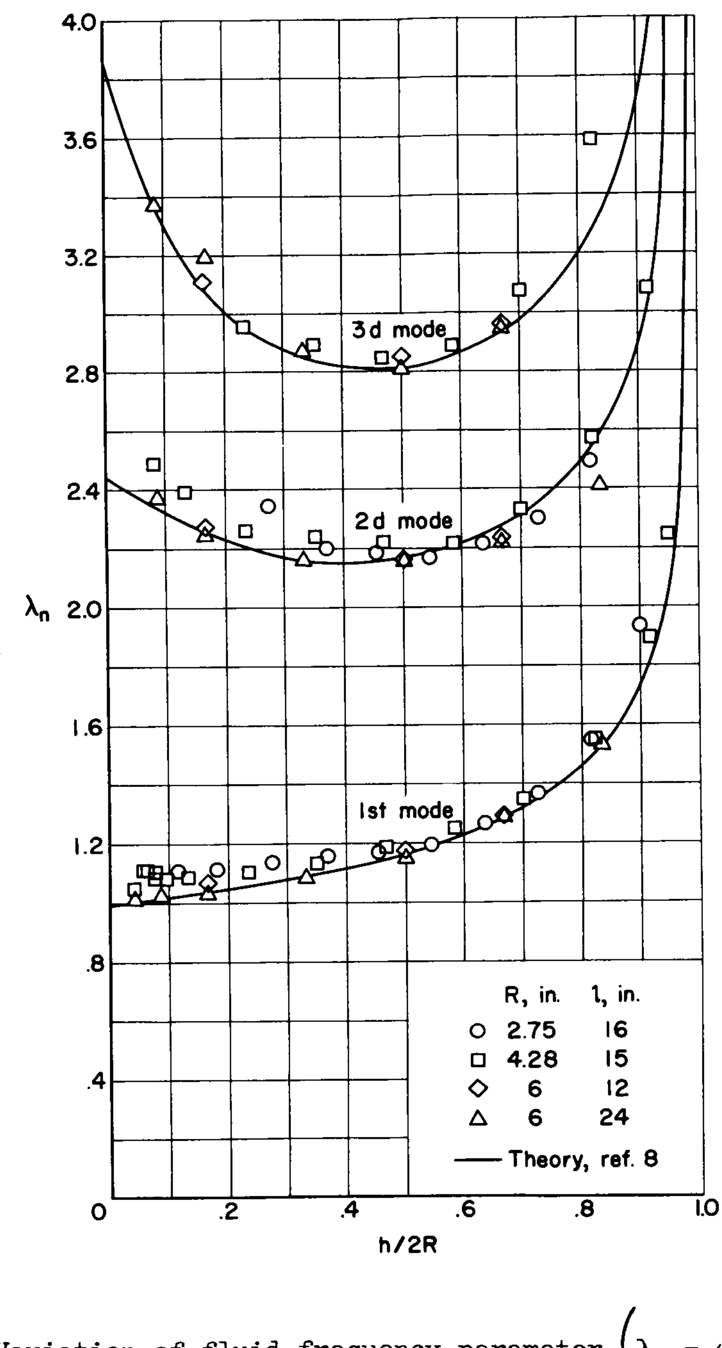

Figure 7.- Variation of fluid frequency parameter $\left(\lambda_n = \omega_n\sqrt{\frac{R}{g}}\right)$ with depth for transverse modes of horizontal circular cylinders.

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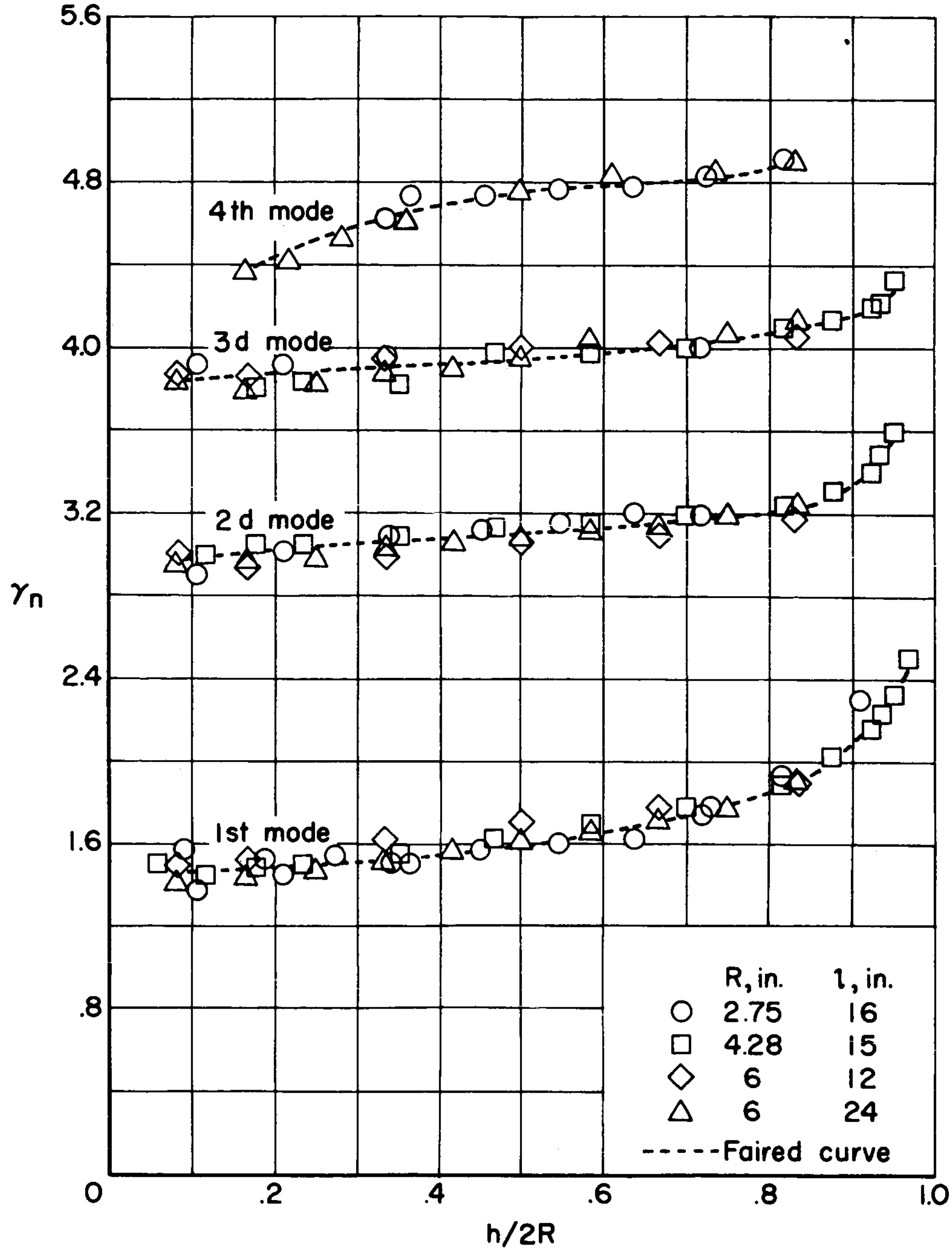

Figure 8.- Variation of fluid frequency parameter $\left(\gamma_n = \omega_n\sqrt{\frac{l}{g}\,\frac{1}{\tanh\frac{n\pi h}{l}}}\right)$ with depth for longitudinal modes of horizontal circular cylinders.

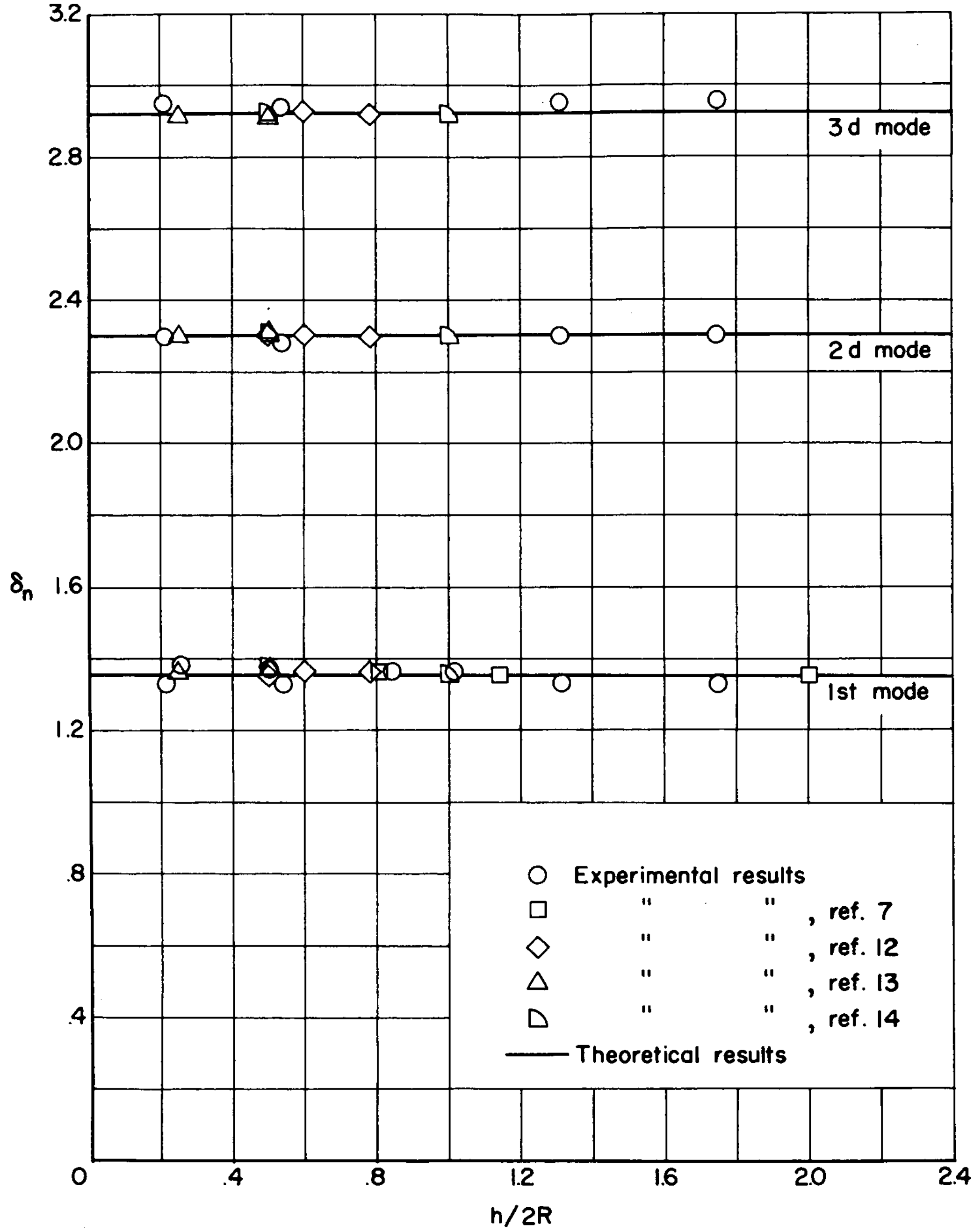

Figure 9.- Variation of fluid frequency parameter $\left(\delta_n = \omega_n\sqrt{\frac{R}{g}\,\frac{1}{\tanh\frac{h}{R}\,\epsilon_n}}\right)$ with depth for upright circular cylinders.

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